



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 23

One Sided Duration

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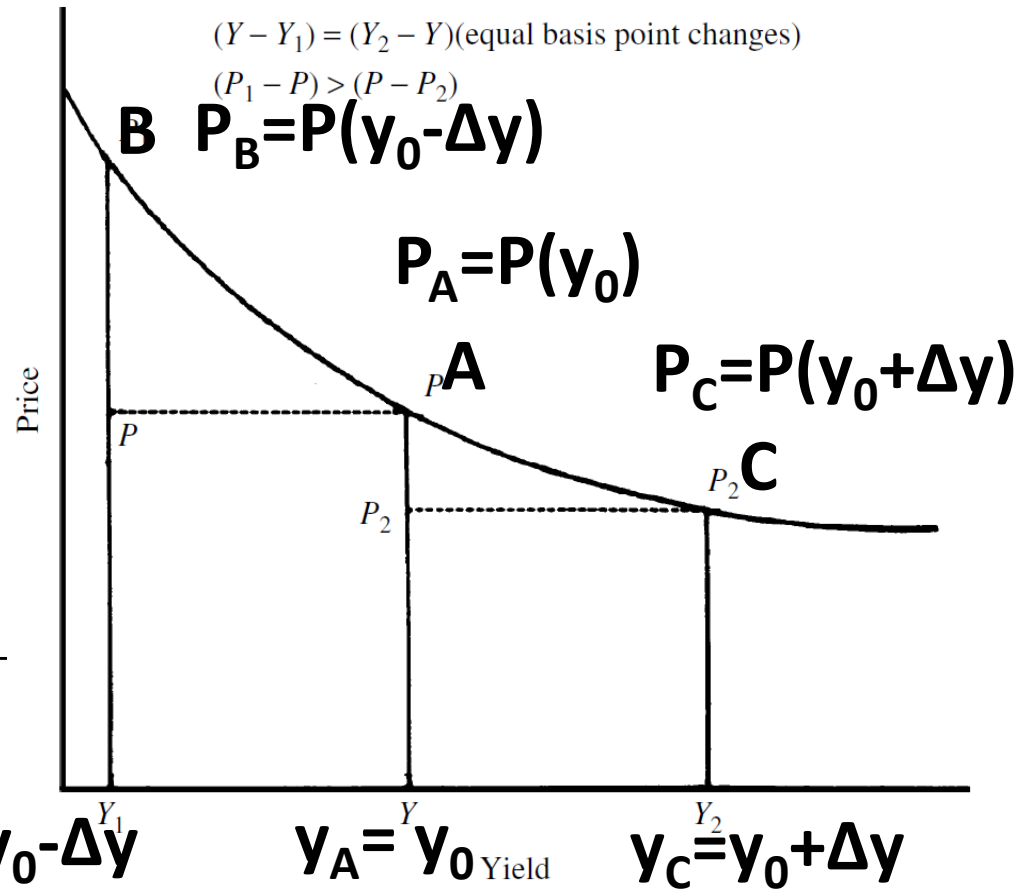


EFFECTIVE/MODIFIED DURATION $D_{\text{eff}}/D_{\text{mod}}$

$$P'(y_0) = \frac{P(y_0 + \Delta y) - P(y_0 - \Delta y)}{\Delta y - (-\Delta y)}$$

$$= \frac{P(y_0 + \Delta y) - P(y_0 - \Delta y)}{2\Delta y} = \frac{P_C - P_B}{(y_C - y_B)}$$

$$D_{\text{Mod}} = (-) \frac{P'(y_0)}{P(y_0)} = (-) \frac{P_C - P_B}{(y_C - y_B) \times P_A}$$



ONE SIDED DURATION

- The computation of effective duration discussed earlier relied on computing the value of the bond for equal parallel shifts of the yield curve Δy up and down (by the same amount).
- This metric captures interest rate risk reasonably well for small changes in the yield curve and for option-free bonds.

- For a callable bond, when the call option is at or near the money, the change in price for a decrease in yield will be less than the change in price for an equal amount of increase in yield.
- Why, because the upper value of a callable bond is capped by its call price.
- Thus, the bond's value will not increase beyond the call price regardless of how low interest rates fall.
- Similarly, the value of a puttable bond is more sensitive to downward movements in yield curve versus upward movements.



- For bonds with embedded options, one-sided durations—durations that apply only when interest rates rise (or, alternatively, only when rates fall)—are better at capturing interest rate sensitivity than simple effective duration.
- When the underlying option is at the-money (or near-the-money), callable bonds will have lower one-sided down duration than one-sided up-duration.



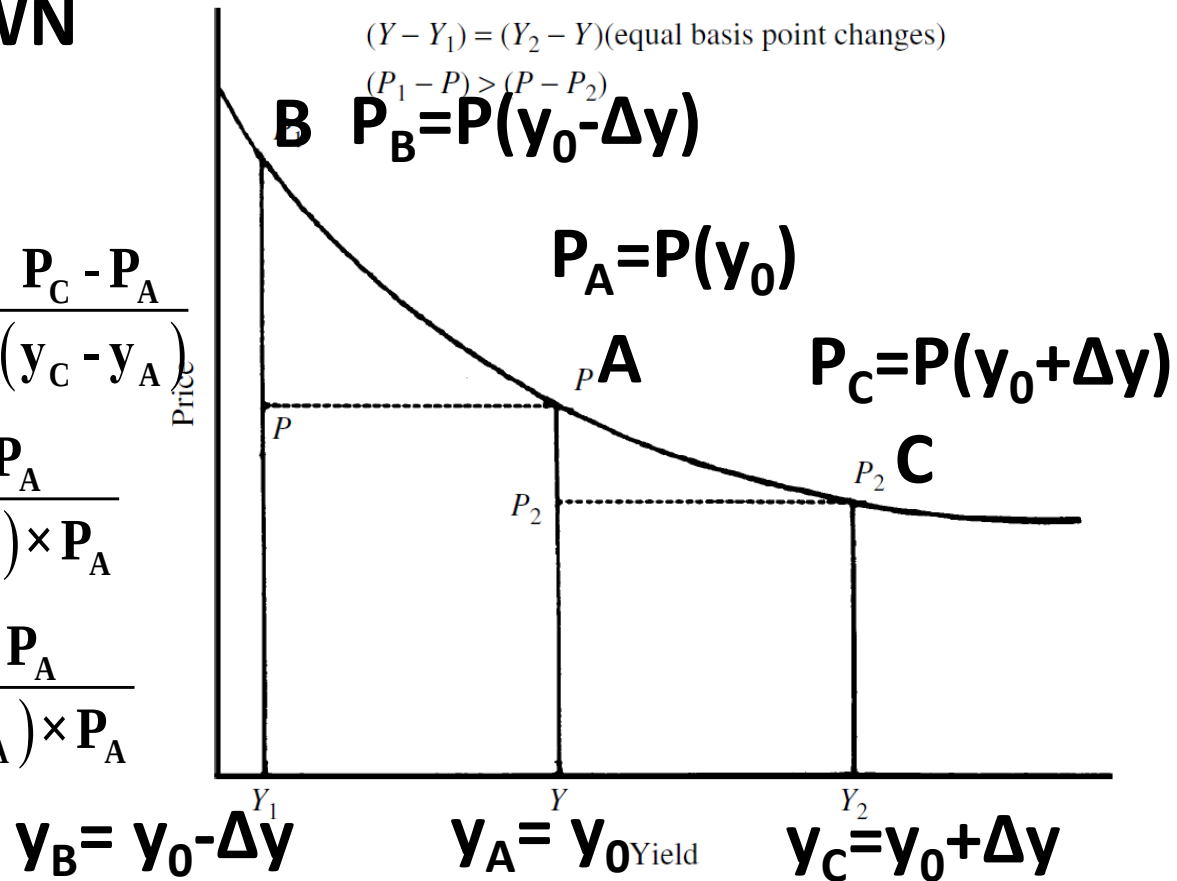
- This is because when interest rates fall, call options will come into play to truncate the value of the bond at the call exercise.
- Thus, the price rise of a callable bond when rates fall is smaller than the price fall for an equal increase in rates.
- Conversely, a near-the money puttable bond will have larger one-sided down-duration than one-sided up-duration.

ONE SIDED UP/DOWN DURATION

$$P'(y_0) = \frac{P(y_0 + \Delta y) - P(y_0)}{\Delta y} = \frac{P_C - P_A}{(y_C - y_A)}$$

$$D_{\text{Mod}}^{\text{Up}} = (-) \frac{P'(y_0)}{P(y_0)} = (-) \frac{P_C - P_A}{(y_C - y_A) \times P_A}$$

$$D_{\text{Mod}}^{\text{Down}} = (-) \frac{P'(y_0)}{P(y_0)} = (-) \frac{P_B - P_A}{(y_B - y_A) \times P_A}$$



COMPARISON OF EFFECTIVE CONVEXITIES OF CALLABLE, PUTTABLE, AND STRAIGHT BONDS

- **Straight bonds have positive effective convexity.**
- **The increase in the value of an option free bond is higher when rates fall than the decrease in value when rates increase by an equal amount.**



- **When rates are high, callable bonds are unlikely to be called and will exhibit positive convexity.**
- **When the underlying call option is near the money, its effective convexity turns negative.**
- **The upside potential of the bond's price is limited due to the call (while the downside is not protected).**
- **Puttable bonds exhibit positive convexity throughout.**

RECAP OF IMPORTANT RESULTS



SPOT & FORWARD RATES

NO ARBITRAGE CONDITION

- $(1 + S_{0T})^T = (1 + S_{01})(1 + f_{12}) \dots (1 + f_{T-1,T})$
- $= (1 + S_{02})^2(1 + f_{23}) \dots (1 + f_{T-1,T})$
- $= (1 + S_{0H})^H(1 + f_{H,H+1}) \dots (1 + f_{T-1,T})$

SPOT BOND PRICES

- $$P_0 = \frac{C_1}{(1+S_{01})^1} + \frac{C_2}{(1+S_{02})^2} + \dots + \frac{C_T}{(1+S_{0T})^T}$$
- $$= \frac{C_1}{(1+S_{01})} + \frac{C_2}{(1+S_{01})(1+f_{12})} + \dots + \frac{C_T}{(1+S_{01})(1+f_{12})\dots(1+f_{T-1,T})}$$

FORWARD BOND PRICES

- $F(0,1,T) = P_0(1 + S_{01}) - C_1$
- $= \frac{C_2}{(1+f_{12})} + \dots + \frac{C_T}{(1+f_{12})\dots(1+f_{T-1,T})}$
- $F(0,2,T) = P_0(1 + S_{02})^2 - C_1(1 + f_{12}) - C_2$
- $= \frac{C_3}{(1+f_{23})} + \dots + \frac{C_T}{(1+f_{23})\dots(1+f_{T-1,T})}$

FORWARD BOND PRICES: ZCBs

- $F(0, 1, T) = P_0(1 + S_{01}) = \frac{C_T}{(1+f_{12})\dots(1+f_{T-1,T})}$
- $F(0, 2, T) = P_0(1 + S_{02})^2 = \frac{C_T}{(1+f_{23})\dots(1+f_{T-1,T})}$

FORWARD YIELDS ON ZCBs

- $y(0, 1, 2) = \frac{F(0,2,T) - F(0,1,T)}{F(0,1,T)}$
- $= \frac{P_0(1+S_{01})(1+f_{12}) - P_0(1+S_{01})}{P_0(1+S_{01})}$
- $= f_{12}$
- Similarly, $y(0, 2, 3) = f_{23}$ etc.
- Thus, the forward yield equals the prevailing forward interest rate.

SLOPE OF THE FORWARD CURVE

- Consider an upward sloping yield curve so that $S_{02} = S_{01} + \Delta$, $\Delta > 0$
- $(1 + S_{02})^2 = (1 + S_{01})(1 + f_{12})$
- $(1 + S_{01} + \Delta)^2 = (1 + S_{01})(1 + f_{12})$ or
- $1 + S_{01}^2 + \Delta^2 + 2S_{01} + 2\Delta + 2S_{01}\Delta = 1 + S_{01} + f_{12} + S_{01}f_{12}$
- $\Delta^2 + 2S_{01} + 2\Delta + 2S_{01}\Delta = S_{01} + f_{12} + S_{01}(f_{12} - S_{01})$
- Ignoring Δ^2 likely to be extremely small, we get $2\Delta(1 + S_{01}) = (f_{12} - S_{01}) + S_{01}(f_{12} - S_{01})$ or $S_{01} + 2\Delta = f_{12}$
- Thus, the change in forward rates is approx. twice the change in spot rates.

ILLUSTRATION

- You are given $S_{01} = 10\%$, $S_{02} = 11\%$.
Calculate f_{12} .
- $$f_{12} = \frac{(1+S_{02})^2}{(1+S_{01})} - 1 = \frac{(1.11)^2}{(1.10)} - 1 = 12\%$$

AMORTIZATION OF PREMIUM & DISCOUNT

- If the yield curve is flat and a bond is initially priced at a premium to par, the projected price at end of period will be lower than start-of-period price as the bond's price is pulled to par at expiration.
- If the yield curve is flat and a bond is initially priced at a discount to par, the projected price at end of period will be higher than start-of-period price as the bond's price is pulled to par at expiration.

ILLUSTRATION

- Assume a 10% 2-year annual coupon bond (FV=1,000) priced at a YTM of 15%. Calculate its current price and forward price at the end of one year assuming the yield curve to be flat.
- We have: $P_0 = \frac{C_1}{(1+y)} + \frac{C_2}{(1+y)^2} = \frac{100}{(1+0.15)} + \frac{1100}{(1+0.15)^2} = 918.72$
- Similarly, $P_1 = \frac{C_2}{(1+y)} = \frac{1100}{(1+0.15)} = 956.52$
- $P_2 = 1100$

PRICING WITH SLOPING YIELD CURVE

- With a sloped yield curve that may not always be true in the shorter run (before maturity).
- If the yield curve is upward sloping, the rolldown return will be higher than the start-of-period YTM.
- For an upward sloping yield curve, the yields at the end of the holding period (based on the term to maturity less the investment horizon) will be lower than the $t=0$ yield which is based on a longer term to maturity.
- Thus, the bond value at the end of the holding period will be priced at a lower YTM than at $t=0$.